A new class of smoothing methods for mathematical programs with equilibrium constraints

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Abstract

A class of smoothing methods is proposed for solving mathematical programs with equilibrium constraints. We introduce new and very simple regularizations of the complementarity constraints. Some estimate distance to optimal solution and expansions of the optimal value function are presented. Limited numerical experiments using SNOPT algorithm are presented to verify the efficiency of our approach.

1 Introduction

Mathematical programs with equilibrium constraints (MPECs) constitute an important class of optimization problems and pose special theoretical and numerical challenges.

MPECs are constrained optimization problems in which the essential constraints are defined by some parametric variational inequalities or a parametric complementarity system. MPECs can be closely related to the well-known Stackelberg game and to general bilevel programming. As a result, MPECs play a very important role in many fields such as engineering design, economic equilibrium, multilevel game, and mathematical programming theory itself, and it has been receiving much attention in the optimization world.

However, MPECs are very difficult to deal with because, the feasible region and optimal solution set are not convex or concave or even connected. Moreover, the constraints can not satisfy any standard constraint qualification such as the linear independence constraint qualification or the Mangasarian-Fromovitz constraint qualification at any feasible point [4, 13].

In this paper, we consider MPECs in their standard complementarity constrained optimization problems formulation

$$\begin{cases} \min & f(x,y) \\ \text{s.t.} & x \in \mathcal{X}, y \in \mathbb{R}^m, z \in \mathbb{R}^l, \lambda \in \mathbb{R}^l, \\ & F(x,y) - \nabla_y g(x,y)^T \lambda = 0 \\ & g(x,y) = z \\ & z \geq 0, \lambda \geq 0, \lambda^T z = 0 \end{cases} \tag{1.1}$$

where the functions $f : \mathbb{R}^{n+m} \to \mathbb{R}$, $F : \mathbb{R}^{n+m} \to \mathbb{R}^m$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}^l$ are all twice continuously differentiable and $\mathcal{X}$ is a nonempty and compact subset of $\mathbb{R}^n$.

Remark. The constraints of (1.1) correspond to the KKT conditions of the parametrized variational inequality

$$y \in C(x) \text{ and } (v-y)^T F(x,y) \geq 0 \text{ for all } v \in C(x), \tag{1.2}$$

where $C(x) := \{y \in \mathbb{R}^m / g(x,y) \geq 0\}$.

The negative properties of MPECs make these problems very difficult and exclude any direct use of standard non linear programming (NLP) algorithms.

In this paper we propose some smoothing techniques to regularize the complementarity constraints and construct relaxed problems that are suitable for NLP algorithms.

Many regularization and relaxation techniques have already been proposed, here is an incomplete list of such methods

$$\begin{array}{lll} (Reg(t)[11,\ 12]) & \lambda^T z = 0 \text{ is relaxed to} & \lambda_i z_i \leq t \quad \forall i \\ (Regeq(t)[11,\ 12]) & \lambda^T z = 0 \text{ is replaced by} & \lambda_i z_i = t \quad \forall i \\ (RegCp(t)[11,\ 12]) & \lambda^T z = 0 \text{ is relaxed to} & \lambda^T z \leq t \\ (Facc.[5]) & \lambda^T z = 0 \text{ is replaced by} & \sqrt{(\lambda_i - z_i)^2 + 4t^2} - (\lambda_i + z_i) = 0\ \forall i \\ (Entro.[2,\ 6]) & \lambda^T z = 0 \text{ is replaced by} & t \ln\{e^{\frac{-\lambda_i}{t}} + e^{\frac{-z_i}{t}}\} = 0 \quad \forall i. \end{array} \tag{1.3}$$

In almost all these techniques, the constraints ($z \geq 0, \lambda \geq 0$ and $\lambda_i z_i = 0$) or $\min(\lambda_i, z_i) = 0$ are replaced by some smooth approximations and maintain the positivity constraints.

In our approach, we maintain the positivity constraints and interpret the complementarity constraint componnent-wise as:

$$\forall i, \quad \text{At most one of } z_i \text{ or } \lambda_i \text{ is nonzero.}$$

So, we construct some parameterized real functions that satisfy:

$$(\theta_r(x) \simeq 1 \text{ if } x \neq 0) \text{ and } (\theta_r(x) \simeq 0 \text{ if } x = 0)$$

to count nonzeros and then replace the constraint

$$\lambda_i z_i = 0$$

by

$$\theta_r(\lambda_i) + \theta_r(z_i) \leq 1.$$

In section 2, we present some preliminaries and assumptions on the problem (1.1) (essentially the same as in [11]) . In Section 3, the smoothing functions and techniques are presented and many approximation and regularity properties are proved. Section 4 is devoted to the analysis of the regularization process. The last section presents some numerical experiments concerning two smoothing functions.

2 Assumptions and preliminaries

We essentially need the same assumptions and background as in [11]. A complete presentation of this background needs about 6 to 7 pages. We will only present in this section some definitions, known optimality conditions and constraint qualifications. For some others we will only refer readers to [11]. These notions will be useful in the next section. The first definition concern a first order optimality condition: the strong stationarity

Definition 2.1 *A feasible point $(x^*, y^*, z^*, \lambda^*)$ is strongly stationary for (1.1) if $d = 0$ solves*

$$\begin{cases} \min & \nabla f(x^*, y^*)^T d_{x,y} \\ \text{s.t.} & d_x \in Add(\mathcal{X}(x^*)), d_z \in \mathbb{R}^l, d_\lambda \in \mathbb{R}^l, \\ & \nabla F(x^*, y^*)^T d_{x,y} - \nabla_y g(x^*, y^*)^T d_\lambda - \nabla(\nabla_y g(x^*, y^*))^T d_{x,y} = 0 \\ & \nabla g(x^*, y^*)^T d_{x,y} - d_z = 0 \\ & (d_z)_i = 0, i \in I_z \backslash I_\lambda \\ & (d_\lambda)_i = 0, i \in I_\lambda \backslash I_z \\ & (d_z)_i \geq 0, (d_\lambda)_i \geq 0, i \in I_z \cap I_\lambda \end{cases} \tag{2.1}$$

where $d = (d_x, d_y, d_z, d_\lambda)^T \in \mathbb{R}^{n+m+2l}$, I_z and I_λ are the active sets at $(x^*, y^*, z^*, \lambda^*)$

$$I_z := \{i = 1, \ldots, l | z_i^* = 0\} \text{ and } I_\lambda := \{i = 1, \ldots, l | \lambda_i^* = 0\}$$

and $Add(\mathcal{X}(x^*))$ is the admissible directions set defined by

$$Add(\mathcal{X}(x^*)) := \{d_x \in \mathbb{R}^m | \exists r_0 > 0 \quad \forall 0 \leq r \leq r_0 \quad x^* + r d_x \in \mathcal{X}.\}$$

Remark. There is an other kind of stationarity (the B-stationarity) which is **less** restrictive but **very difficult to chek**. We prefer to not present it in this paper. These two stationarity properties are equivalent when the MPEC-LICQ (defined next) is satisfied

Definition 2.2 *The MPEC-LICQ is satisfied at the point* $(x^*, y^*, z^*, \lambda^*)$ *if the linear independance constraint qualification (LICQ) is satisfied for the following RNLP problem at* $(x^*, y^*, z^*, \lambda^*)$.

$$\begin{cases} \min & f(x,y) \\ \text{s.t.} & x \in \mathcal{X}, z \in \mathbb{R}^l, \lambda \in \mathbb{R}^l, \\ & F(x,y) - \nabla_y g(x,y)^T \lambda = 0 \\ & g(x,y) = z \\ & z_i = 0, i \in I_z \backslash I_\lambda \\ & \lambda_i = 0, i \in I_\lambda \backslash I_z \\ & z_i \geq 0, \lambda_i \geq 0, i \in I_z \cap I_\lambda \end{cases} \tag{2.2}$$

An other important and usefull constraint qualification is the following Mangasarian-Fromovitz one

Definition 2.3 *The MPEC-MFCQ is satisfied at the point* $(x^*, y^*, z^*, \lambda^*)$ *if the Mangasarian-Fromovitz constraint qualification (MFCQ) is satisfied for the RNLP problem at* $(x^*, y^*, z^*, \lambda^*)$.

We will also use some Second-Order sufficient conditions namely: the (MPEC-SOSC) and the (RNLP-SOSC). These two conditions (among others) are defined in [11].

3 The smoothing technique

For $r > 0$, we consider real functions $\theta_r : \mathbb{R}_+ \to [0,1]$ satisfying

$$\begin{array}{ll} (i) & \theta_r \text{ is nondecreasing, strictly concave and continuously differentiable,} \\ (ii) & \forall r > 0, \quad \theta_r(0) = 0, \\ (iii) & \forall x > 0, \quad \lim_{r \to 0} \theta_r(x) = 1, \text{ and} \\ (iv) & \lim_{r \to 0} \theta'_r(0) > 0. \end{array} \tag{3.1}$$

We will present some interesting examples of such functions after the following approximation result

Lemma 3.1 *For any* $\varepsilon > 0$, *and* $x, y \geq 0$, *there exists* $r_0 > 0$ *such that*

$$\forall r \leq r_0, \qquad (\min(x,y) = 0) \Longrightarrow (\theta_r(x) + \theta_r(y) \leq 1) \Longrightarrow (\min(x,y) \leq \varepsilon).$$

Proof - The first property is obvious since $\theta_r(0) = 0$ and $\theta_r \leq 1$.
Using assumption (iii) for $x = \varepsilon$, we have

$$\forall \alpha > 0, \quad \exists r_0 > 0 / \quad \forall r \leq r_0 \qquad 1 - \theta_r(\varepsilon) < \alpha,$$

so that, if we suppose that $\min(x,y) > \varepsilon$, assumption (i) gives

$$\theta_r(x) + \theta_r(y) > 2\theta_r(\varepsilon) > 2(1-\alpha).$$

Then if we choose $\alpha < \frac{1}{2}$, we obtain that $\theta_r(x) + \theta_r(y) > 1$. $\square$

This first approximation result can be improved for some interesting choices of the smoothing functions θ_r

$$\begin{array}{lll} (\theta_.^1) & \theta_r^1(x) = \dfrac{x}{x+r} & \\ (\theta_.^{W_k}) & \theta_r^{W_k}(x) = 1 - e^{-(\frac{x}{r})^k} & \text{for } k > 0 \\ (\theta_.^{\log}) & \theta_r^{\log}(x) = \dfrac{\log(1+x)}{\log(1+x+r)} & \end{array}$$

We will also consider the general class $\Theta^{\geq 1}$ of functions

$$(\theta_.^{\geq 1}) \qquad \text{verifying (i} - \text{iv) and } \theta_.^{\geq 1} \geq \theta_.^1$$

Remark. The functions $\theta_.^{W_k}$ are the density functions of Weibull distributions, when $k = 1$, the obtained smoothing method corresponds (with slight modifications) to the inequality entropic regularization [2]. Simple comparison calculus proove that $\theta_.^{\log}$ and $\theta_.^{W_k}$ for $(0 < k \leq 1)$ belong to the class of functions $\Theta^{\geq 1}$.

Lemma 3.2 *we have*

$$\begin{array}{lll} (i) & \forall x \geq 0,\ \forall y \geq 0 & \theta_r^1(x) + \theta_r^1(y) \leq 1 \Longleftrightarrow x \cdot y \leq r^2, \textit{and} \\ (ii) & \forall x \geq 0,\ \forall y \geq 0 & x \cdot y = 0 \Longrightarrow \theta_r^{\geq 1}(x) + \theta_r^{\geq 1}(y) \leq 1 \Longrightarrow x \cdot y \leq r^2. \end{array}$$

Proof - (i) We have

$$\theta_r^1(x) + \theta_r^1(y) = \frac{2xy + rx + ry}{xy + rx + ry + r^2},$$

so that

$$\begin{aligned} \theta_r^1(x) + \theta_r^1(y) \leq 1 &\Longleftrightarrow 2xy + rx + ry \leq xy + rx + ry + r^2 \\ &\Longleftrightarrow x \cdot y \leq r^2. \end{aligned}$$

The first part of (ii) follows obviously from Lemma 3.1 and the second one is a direct consequence of (i) since

$$\theta_r^{\geq 1}(x) + \theta_r^{\geq 1}(y) \leq 1 \Longrightarrow \theta_r^1(x) + \theta_r^1(y) \leq 1.$$

$\square$

Using any function θ_r satisfying (3.1), we obtain the relaxed following problem for (1.1)

$$\left\{ \begin{array}{ll} \min & f(x,y) \\ \text{s.t.} & (x,y) \in \mathcal{X}, z \in \mathbb{R}_+^l, \lambda \in \mathbb{R}_+^l, e \in \mathbb{R}_+^l \\ & F(x,y) - \nabla_y g(x,y)^T \lambda = 0 \\ & g(x,y) = z \\ & \theta_r(\lambda_i) + \theta_r(z_i) + e_i = 1, \quad \forall i \in \{1, ..., l\}. \end{array} \right. \tag{3.2}$$

Remarks. (i) By choosing some particular smoothing functions (ex. $\theta_r^{W_k}$), the nonnegativity constraints on λ and z become implicite and can be removed from the definition of (3.2).(This can have an important impact in practice.)
(ii) Under some classical assumptions, as in [5] we can easily prove that the jacobian of equality constraints (with respect to (y, z, λ) is nonsingular. This property is useful in practice since standard NLP algorithms use Newton-type to solve systems of nonlinear equations corresponding to this jacobian.

Lemma 3.3 *If g is concave with respect to y and F is uniformly strongly monotone with respect to y, then for every nonnegative r and every feasible point (x, y, z, λ, e) of problem (3.2), the jacobian of equality constraints (with respect to (y, z, λ)) is nonsingular.*

Proof - Using the assumptions 3.1 (i) and (iv), the proof is exactely the same as in [2] or [5]. □

Problem (3.2) may be viewed as a perturbation of (1.1). Previous lemmas prove that (3.2) is in fact some tight relaxation of (1.1). However this perturbation is not continuous on the parameter r so that any direct use of perturbation results such that [3] is impossible.
Fortunately, Lemma 3.2 proves that for the particular smoothing function $\theta_.^1$, the corresponding relaxed problem (3.2) is equivalent to ($Reg(\mathbf{r^2})$) in [11]. We can then benefit from the theoretical results in [11].

The following results provide, in the case of the $\theta_.^1$ function, some distance estimate between solution of (3.2) and solution of (1.1). These results correspond to applications of [[3], Theorem 5.57, Theorem 4.55 and Lemma 4.57] and can be found with complete proofs in [11]. We just state them in our context and add the optimal value expansion.

Theorem 3.1 *Suppose that $X^* = (x^*, y^*, z^*, \lambda^*)$ is a strongly stationary point of (1.1) at which MPEC-MFCQ and MPEC-SOSC are satisfied. Then there are positive constants α, $\bar{r}$, and M such that for all $r \in (0, \bar{r}]$, the global solution $X(r)$ of the localized problem (3.2) with the additional ball constraint $\|X - X^*\| \leq \alpha$ that lies closest to X^* satisfies $\|X(r) - X^*\| \leq M.r$. Furthermore the optimal value v_r of (3.2) has an expansion of the form*

$$v_r = v^0 + \frac{1}{2}.a.r^2 + o(r^2)$$

where v^0 is the optimal value of (1.1) and a is the optimal value of an auxiliary quadratic problem[3].

Theorem 3.2 *Suppose that $X^* = (x^*, y^*, z^*, \lambda^*)$ is a strongly stationary point of (1.1) at which MPEC-LICQ and RNLP-SOSC are satisfied. Then there are positive constants α, $\bar{r}$, and M such that for all $r \in (0, \bar{r}]$, the global solution $X(r)$ of the localized problem (3.2) with the additional ball constraint $\|X - X^*\| \leq \alpha$ that lies closest to X^* satisfies $\|X(r) - X^*\| \leq M.r^2$. Furthermore the optimal value v_r of (3.2) has an expansion of the form*

$$v_r \leq v^0 + b.r^2 + O(r^4)$$

where v^0 is the optimal value of (1.1) and b is the optimal value of an auxiliary linearized problem[3].

For functions of the general class $\Theta^{\geq 1}$, the corresponding feasible sets satisfy

$$\mathcal{F}_P \subset \mathcal{F}_{\theta^{\geq 1}_{\cdot}} \subset \mathcal{F}_{\theta^1_{\cdot}}$$

where $\mathcal{F}_P$, $\mathcal{F}_{\theta^{\geq 1}_{\cdot}}$ and $\mathcal{F}_{\theta^1_{\cdot}}$ are respectively the feasible set of problem (1.1) and (3.2) for the corresponding θ_r function.
These inclusions prove that the optimal value expansions given in Theorem 3.1 and Theorem 3.2 are still valid under the same assumptions.

Theorem 3.3 *When using functions $\theta^{\geq 1}_{\cdot}$, under the same assumptions of Theorem3.1 (resp. Theorem3.2) the optimal value v_r of (3.2) has an expansion of the form*

$$v_r \leq v^0 + \frac{1}{2}.a.r^2 + o(r^2) \quad \text{(resp.} \quad v_r \leq v^0 + b.r^2 + O(r^4))$$

4 Numerical resuts

For two different smoothing functions, we present some numerical results using the SNOPT [8] nonlinear programming algorithm on the AMPL [1] optimization plateform. Our aim is just to verify the qualitative numerical efficiency of our approach. We consider a subset of the MACMPEC [9] test problems with known optimal values and solutions (we consider the same test problems as in [2, 5]) .
We choose the two functions

$$\theta_r^1(x) = \frac{x}{x+r}$$

and

$$\theta_r^{W_1}(x) = 1 - e^{-\frac{x}{r}}.$$

The first function has (in our analysis) the best theoretical results and corresponds ¨in some way¨to the regularization studied in [12, 11]. While the second one corresponds to the enropic regularization [2, 6].

In our experiments, we made a logarithmic scaling for these two functions to bound their gradients. Each constraint

$$\theta_r(\lambda_i) + \theta_r(z_i) + e_i = 1$$

is in fact replaced by the following inequality

$$r^2 \ln \left(\frac{r}{\lambda_i + r} + \frac{r}{z_i + r} \right) \geq 0,$$

in the case of the θ_r^1 function and

$$r \ln \left(e^{-\frac{\lambda_i}{r}} + e^{-\frac{z_i}{r}} \right) \geq 0.$$

in the case of the $\theta_r^{W_1}$ function.

This scaling technique is used in [2]. It does not make any real change in the number of outer iterations but reduces significantly the total number of minor iterations and evaluations of the objective function, objective gradient, constraints and constraints gradient.

The regularization parameter is chosen the largest possible: we decrease r as long as there is an improvement of the objective function or the constraints.

The two following tables give for each considered problem and for different starting points, the final value of the parameter r, the optimal value and solution obtained when using each of the two smoothing functions. The tables report also different informations concerning the computational effort of the solver SNOPT. *itM* and *itm* correspond to the total number of major and minor iterations numbers [8]. The total number of objective function evaluations is given in (*Obj.*). (*grad.*) corresponds to the total number of objective function gradient evaluations. (*constr.*) and (*jac.*) give respectively the total number of constraints and constraints gradient evaluations.

Problem	r	Start	Obj.val.	Opt.x	(itM,itm)	Obj.	grad	constr.	Jac
Bard1	1.e-2	no	17	(1,0)	(5,8)	9	8	9	8
Df1	1.e-3	no	0	(1,0)	(1,1)	3	2	3	2
Gauvin	1.e-2	no	20	(2,14)	(4,11)	7	6	7	6
jr1	1.e-2	no	0.5	(0.5,0.5)	(6,3)	9	8	9	8
Gnash10	1.e-5	gnash10.dat	-230.8232	47.036	(17,46)	21	20	21	20
Gnash11	1.e-4	gnash11.dat	-129.9119	34.9942	(20,50)	18	17	21	20
Gnash12	1.e-4	gnash12.dat	-36.93311	18.1332	(24,51)	27	26	27	26
Gnash13	1.e-2	gnash13.dat	-7.061783	7.55197	(14,56)	20	19	23	22
Gnash14	1.e-3	gnash14.dat	-0.179046	1.06632	(14,46)	18	17	21	20
Scholtes1	1.e-1	1	2	0	(9,10)	14	13	14	13
Bilevel1	1.e-2	(25,25)	5	(25,30)	(3,11)	0	0	9	8
		(50,50)	5	(25,30)	(0,6)	0	0	2	1
Nash1	1.e-1	(0,0)	1.61e-14	(9.996,4.999)	(13,42)	25	24	25	24
		(5,5)	1.60e-18	(9.313,5.686)	(10,33)	32	31	32	31
		(10,10)	1.46e-14	(9.092,5.901)	(16,38)	34	33	34	33
		(10,0)	3.56e-24	(9.999,4.999)	(12,34)	28	27	28	27
		(0,10)	9.03e-22	(9.999,4.999)	(14,41)	31	30	31	30
Bilevel2	1.e-4	(0,0,0,0)	-6600	(6.441,4.863,12.559,16.137)	(6,43)	9	8	9	8
		(0,5,0,20)	-6600	(6.575,5,12.425,16)	(6,50)	9	8	9	8
		(5,0,15,10)	-6600	(6.837,12.162,16)	(5,36)	7	6	7	6
		(5,5,15,15)	-6600	(4.892,3.373,14.107,17.627)	(5,35)	7	6	7	6
		(10,5,15,10)	-6600	(8.014,4.971,10.986,16.029)	(5,38)	7	6	7	6
Bilevel3	1.e-4	(0,0)	-12.6787	(0,2)	(9,23)	12	11	12	11
		(0,2)	-12.6787	(0,2)	(15,27)	32	31	32	31
		(2,0)	-10.36	(2,0)	(01,06)	3	2	3	2
desilva	1.e-3	(0,0)	-1	(0.5,0.5)	(4,11)	6	5	6	5
		(2,2)	-1	(0.5,0.5)	(3,9)	5	4	5	4
Stack.1	1.e-2	0	-3266.6666	93.3333	(4,9)	6	5	6	5
		100	-3266.6666	93.3333	(3,3)	5	4	5	4
		200	-3266.6666	93.3333	(7,5)	19	18	19	18

Table1: using the θ_r^1 smoothing function

Problem	r	Start	Obj.val.	Opt.x	(itM,itm)	Obj.	grad	constr.	Jac
Bard1	1.e-2	no	17	(1,0)	(13,8)	16	15	16	15
Df1	1.e-3	no	0	(1,0)	(1,2)	3	2	3	2
Gauvin	1.e-2	no	20	(2,14)	(5,12)	7	6	7	6
jr1	1.e-2	no	0.5	(0.5,0.5)	(13,4)	16	15	16	15
Gnash10	1.e-3	gnash10.dat	-230.8232	47.036	(17,63)	19	18	21	20
Gnash11	1.e-3	gnash11.dat	-129.9119	34.9942	(15,48)	18	17	18	17
Gnash12	1.e-1	gnash12.dat	-36.93311	18.1332	(15,43)	19	18	19	18
Gnash13	1.e-1	gnash13.dat	-7.061783	7.55197	(23,69)	30	29	30	29
Gnash14	1.e-3	gnash14.dat	-0.179046	1.06633	(22,38)	27	26	27	26
Scholtes1	1.e-1	1	2	0	(11,11)	16	15	16	15
Bilevel1	1.e-2	(25,25)	5	(25,30)	(3,11)	0	0	9	8
		(50,50)	5	(25,30)	(0,6)	0	0	2	1
Nash1	1.e-1	(0,0)	7.27e-14	(9,6)	(9,16)	12	11	12	11
		(5,5)	4.25e-18	(10,5)	(6,16)	10	9	10	9
		(10,10)	1.09e-11	(9,6)	(13,25)	21	20	21	20
		(10,0)	1.27e-13	(9.355,5.645)	(16,34)	27	26	27	26
		(0,10)	3.29e-15	(9.396,5.604)	(7,16)	11	10	11	10
Bilevel2	1.e-1	(0,0,0,0)	-6600	(4.851,5,14.149,16)	(6,34)	8	7	8	7
		(0,5,0,20)	-6600	(5.195,5,13.805,16)	(5,40)	7	6	7	6
		(5,0,15,10)	-6600	(6.099,4.834,12.901,16.166)	(5,42)	7	6	7	6
		(5,5,15,15)	-6600	(4,1.714,15,19.286)	(5,45)	7	6	7	6
		(10,5,15,10)	-6600	(7.724,5,11.276,16)	(5,50)	7	6	7	6
Bilevel3	1.e-1	(0,0)	-12.6787	(0,2)	(22,38)	29	28	29	28
		(0,2)	-12.6787	(0,2)	(34,61)	56	55	56	55
		(2,0)	-10.36	(2,0)	(01,06)	3	2	3	2
desilva	1.e-2	(0,0)	-1	(0.5,0.5)	(5,9)	8	7	8	7
		(2,2)	-1	(0.5,0.5)	(6,14)	9	8	9	8
Stack.1	1.e-2	0	-3266.6666	93.3333	(4,4)	6	5	6	5
		100	-3266.6666	93.3333	(3,5)	5	4	5	4
		200	-3266.6666	93.3333	(11,4)	14	13	14	13

Table2: using the inequality enropic approach ($\theta_r^{W_1}$)

5 Conclusion

We introduced a new regularization scheme for mathematical programs with complementarity constrains. Our approach is very simple and quite different from existing techniques for the same class of problems. The obtained regularized problems are now suitable for standard NLP algorithms. These regularizations have different theoretical sensivity and regularity properties. The limited numerical experiments give very promising results (comparable to those of [2]) and suggest to make real investigations on functions of the class $\theta_{\cdot}^{W_k}$. Therefore, we hope that some of our smoothing functions will correspond to simple and efficient algorithms for the solution of real-world MPECs and Bilevel programs.

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